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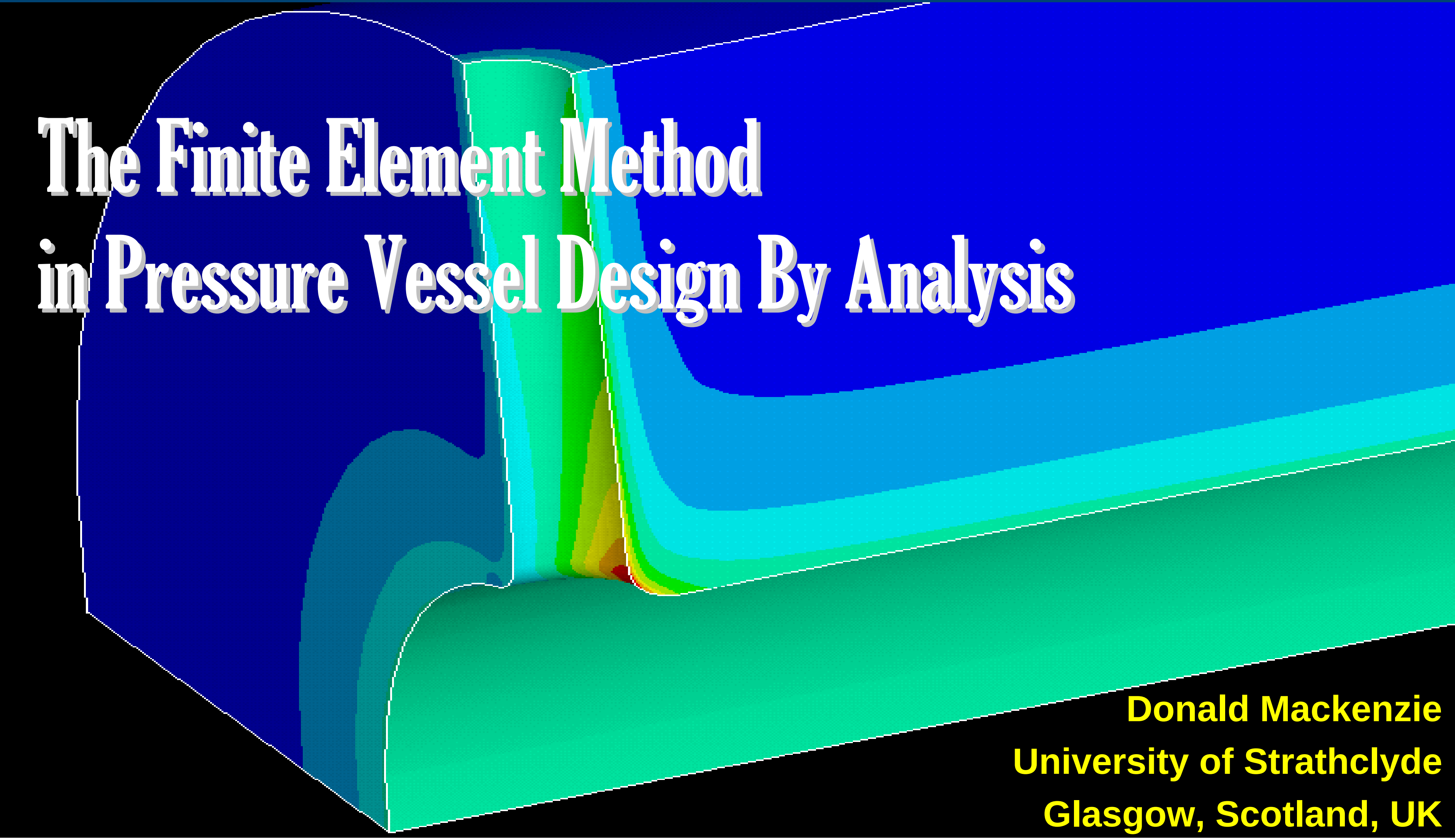

The Finite Element Method in Pressure Vessel Design By Analysis

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FENET THEMATIC NETWORK
COMPETITIVE AND SUSTAINABLE GROWTH
(GROWTH) PROGRAMME

NAFEMS
The International Association for the Engineering Analysis Community

Introduction

- Design by Analysis has an established history in Pressure Vessel Design

- Procedures are specified in national and international codes of practice

    - *UK: PD5500 Unfired Fusion Welded Pressure Vessels*
    - *EU: EN13445: Unfired Pressure Vessels*
    - *USA: ASME Boiler & Pressure Vessel Code Sections III & VIII*

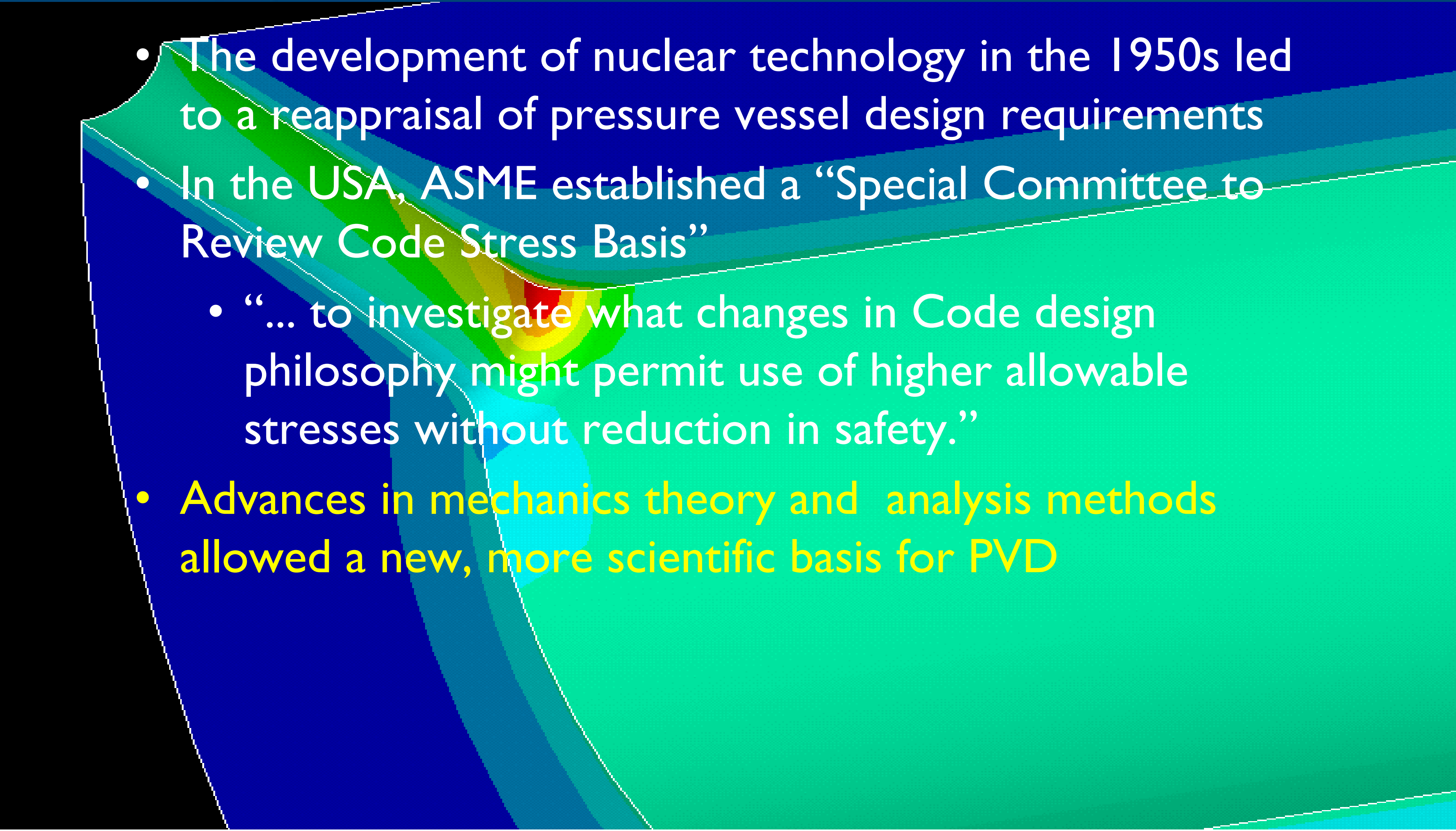
Design by Formula

- Prior to 1963 all Code vessels were designed according to a systematic Design by Formula approach
  - Based on experience and simple mechanics
    - Keep hoop stress low with respect to yield
    - Use ductile material to accommodate local peak stress

- In DBF, the vessel geometry and major dimensions such as radius, length etc. are specified
  - The required thickness is then calculated for a given load using
    - Closed-form equations
    - Graphical data

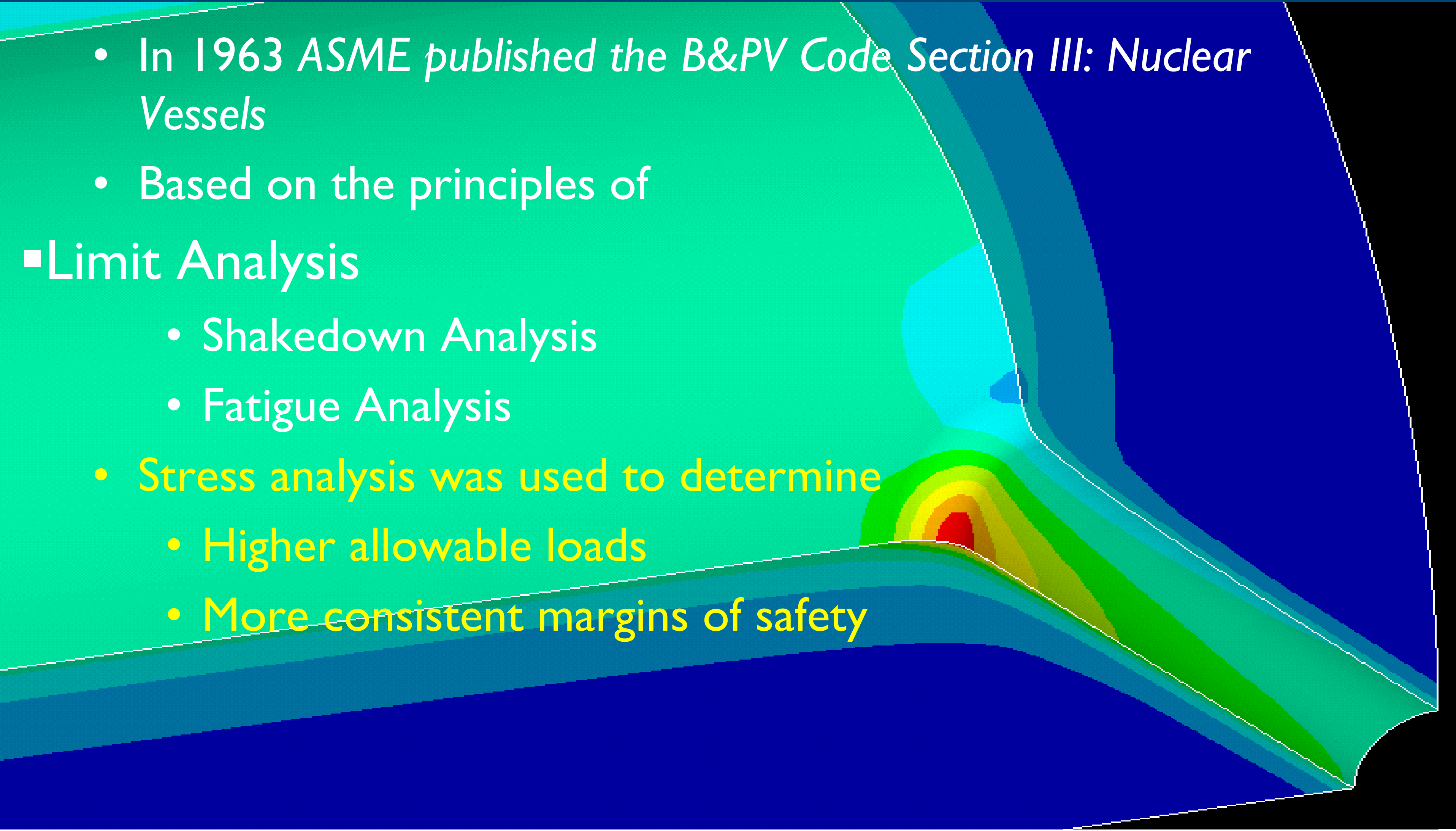
$$t > \frac{pD}{2f+p}$$

Improved Design



- The development of nuclear technology in the 1950s led to a reappraisal of pressure vessel design requirements
- In the USA, ASME established a “Special Committee to Review Code Stress Basis”
  - “... to investigate what changes in Code design philosophy might permit use of higher allowable stresses without reduction in safety.”
- Advances in mechanics theory and analysis methods allowed a new, more scientific basis for PVD

ASME Design Philosophy

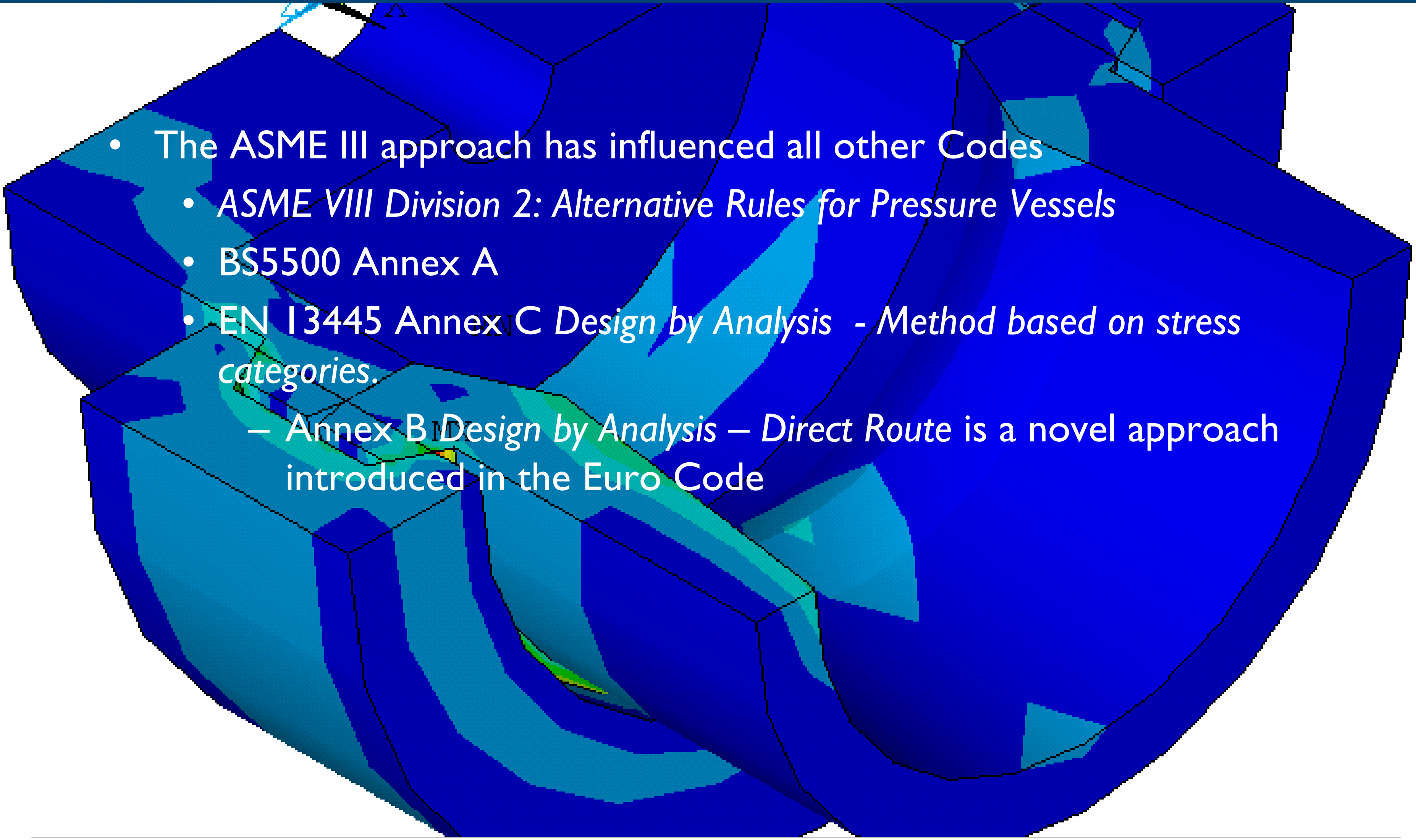


- In 1963 *ASME published the B&PV Code Section III: Nuclear Vessels*
- Based on the principles of

▪ Limit Analysis
  - Shakedown Analysis
  - Fatigue Analysis
- Stress analysis was used to determine
  - Higher allowable loads
  - More consistent margins of safety

- The new Code permitted two approaches to design
  - Improved Design by Formula
    - More accurate formulae for sizing common components
    - Higher allowable stresses
      - Intended for standard configurations
  - Design by Analysis
    - Designer performs stress analysis and evaluates results against Code limits
    - Intended for configurations not covered by DBF

DBA Today



- The ASME III approach has influenced all other Codes
  - *ASME VIII Division 2: Alternative Rules for Pressure Vessels*
  - BS5500 Annex A
  - EN 13445 Annex C *Design by Analysis - Method based on stress categories.*
    - Annex B *Design by Analysis – Direct Route* is a novel approach introduced in the Euro Code

Scope and Purpose of DBA

- PD5500 Annex A
  - "…gives design criteria for stress systems resulting from the application of loads and/or the use of components or types of component not covered explicitly by Section 3 ….. The intention is to ensure … the design basis is consistent [with DBF]… Formal analysis … is only required in the case of significant additional loadings or loadings from components significantly different to those covered in Section 3."
    - Suggests DBA is specifically for vessels outside the DBF remit
    - Does not explicitly state the procedures cannot be applied to vessels covered by DBF

Scope and Purpose of DBA

- EN 13445
  - Annex B.1
    - "Design by analysis provides rules for the design of any component under any action. It may be used … as an alternative to design by formulas … as a compliment to design-by-formulas …"
  - Annex C.1
    - "…may be used … as an alternative to design by formulas … as a compliment to design-by-formulas … as an alternative to the design-by-analysis direct route

Scope and Purpose of DBA

- ASME VIII
  - DBA applies to non-standard configurations
  - The thickness of elements of the vessel covered by DBF rules

    e.g. Cylindrical shell

    Flat head

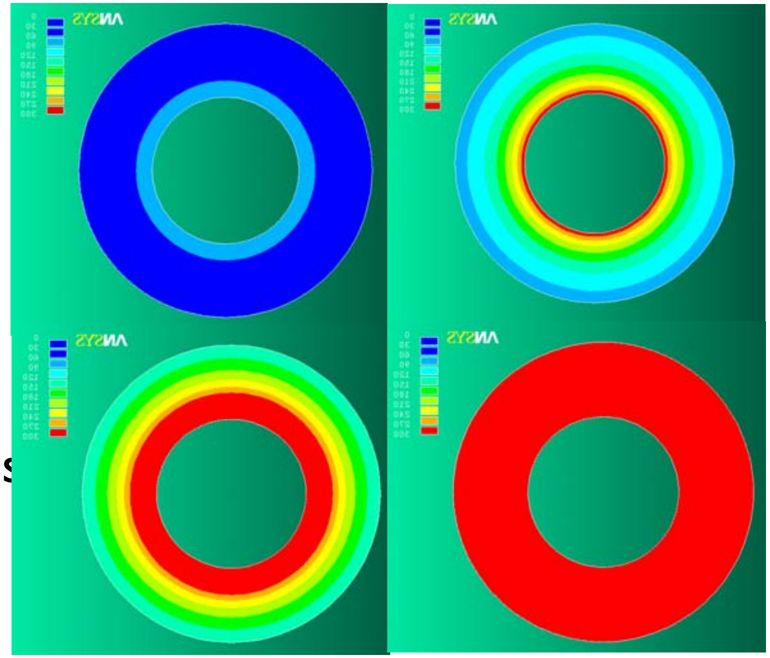
    must meet DBF minimum thickness requirements

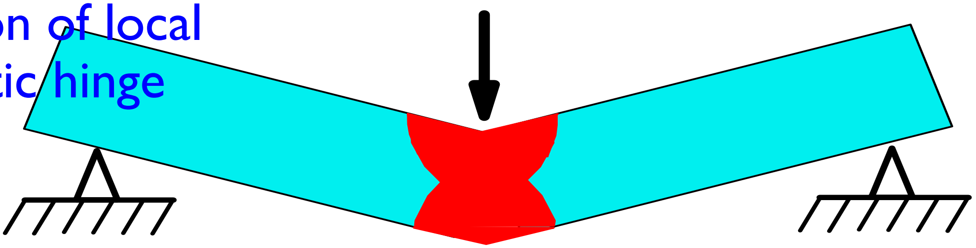
DBA Design Objectives

- The main guidelines prevent
  - Gross plastic deformation or ductile burst under static load
  - Incremental plastic collapse under repeated or cyclic load
  - Fatigue under cyclic load
- Other mechanisms considered as required:
  - Elastic buckling, creep, brittle fracture, stress corrosion, etc.

Gross Plastic Deformation

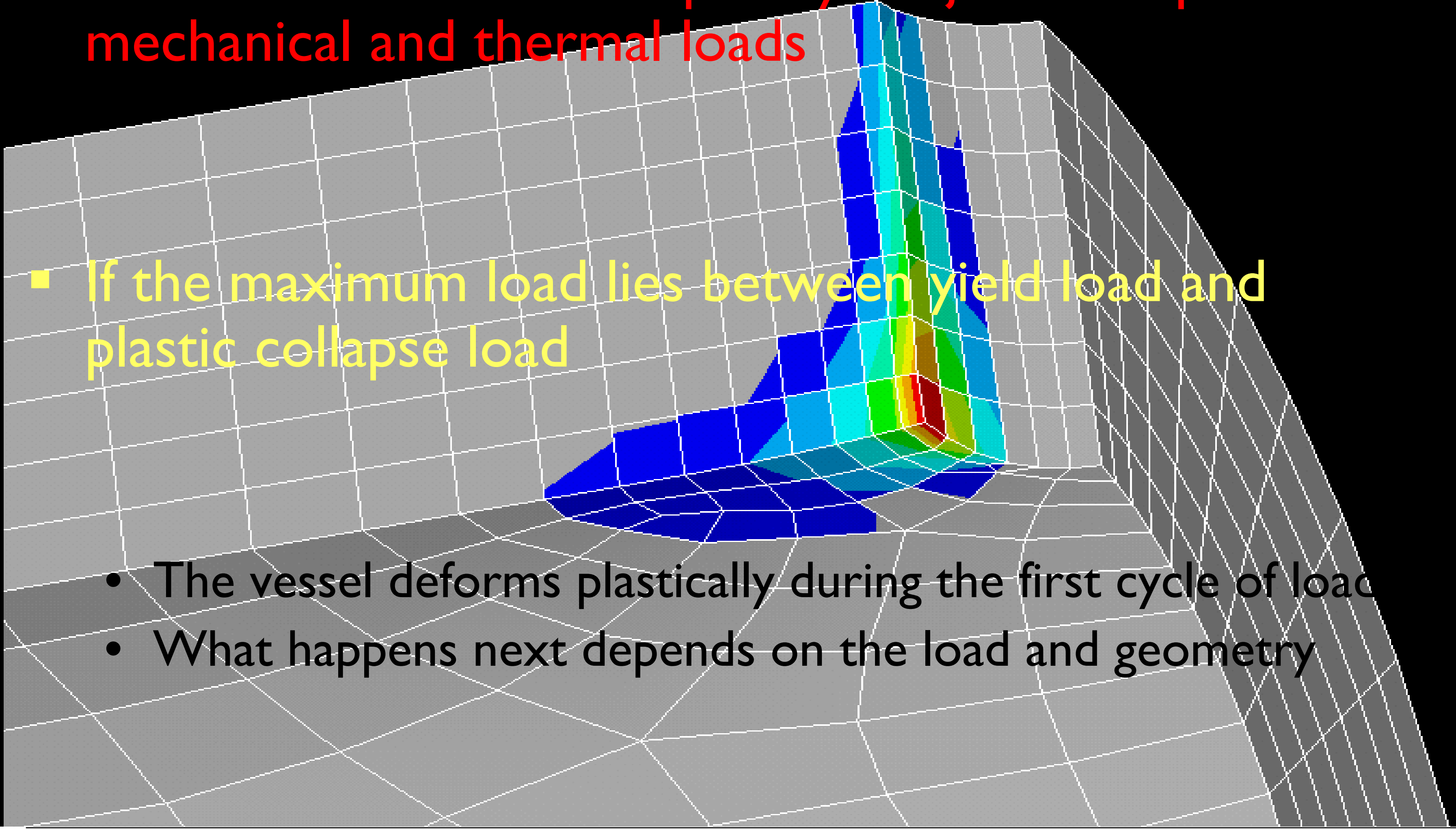
- Associated with ductile failure of the vessel under static mechanical load
- Mechanisms
  - Ductile rupture or collapse when the entire volume of the material experiences plastic deformation
  - Plastic collapse through formation of local areas of plasticity that form plastic hinge mechanisms





Plastic hinge

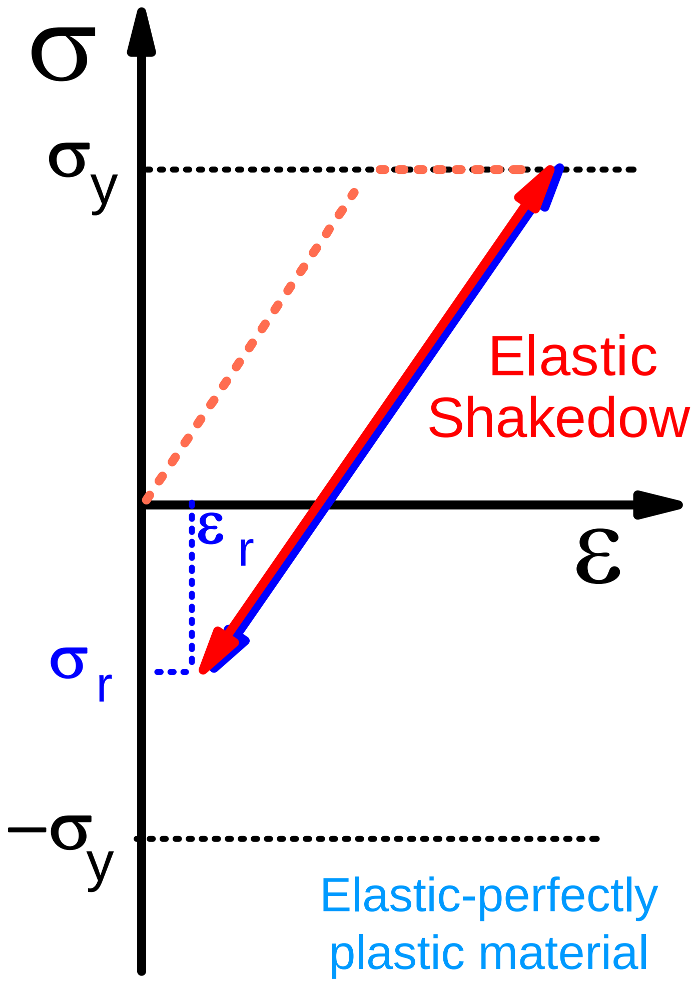
Incremental Plastic Collapse



- Pressure vessels are frequently subject to repeated mechanical and thermal loads
- If the maximum load lies between yield load and plastic collapse load
  - The vessel deforms plastically during the first cycle of load
  - What happens next depends on the load and geometry

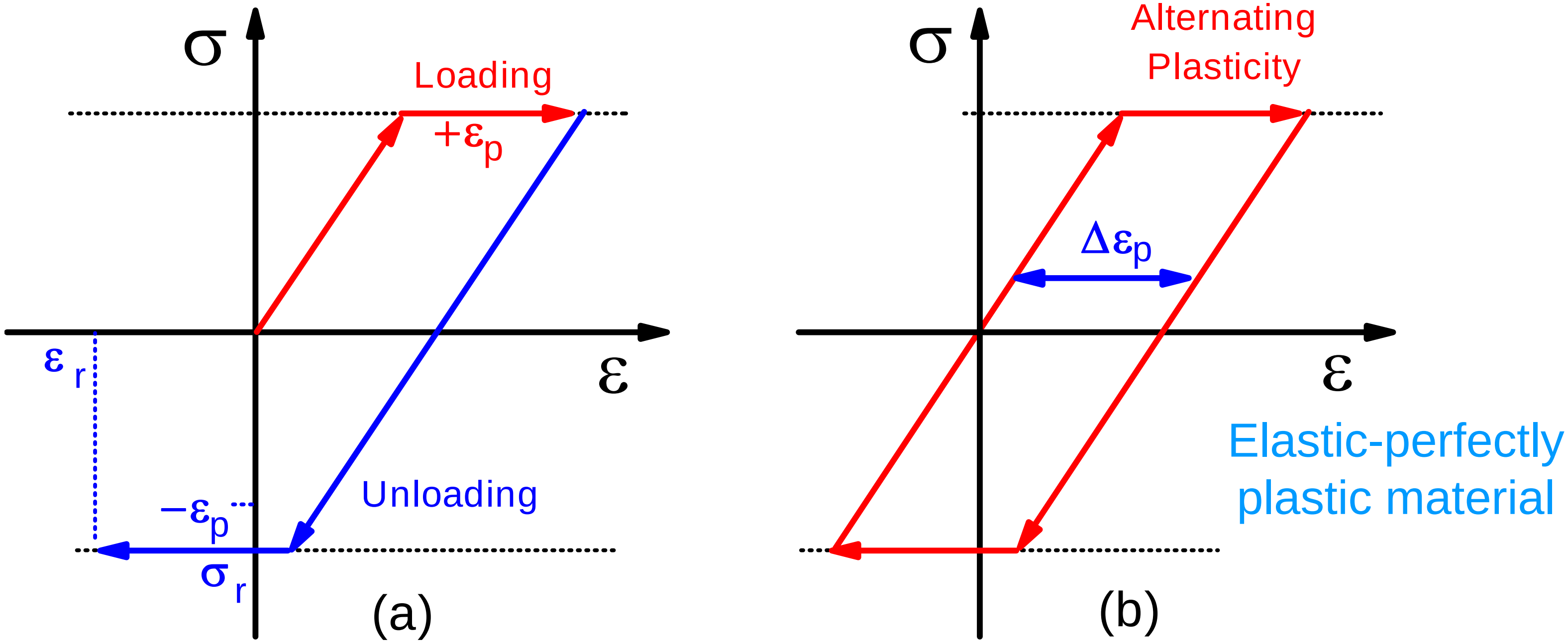
Elastic Shakedown

- If yielding does not occur during unloading subsequent re-loading is wholly elastic
  - The structure is said to exhibit Elastic Shakedown



Elastic-perfectly plastic material

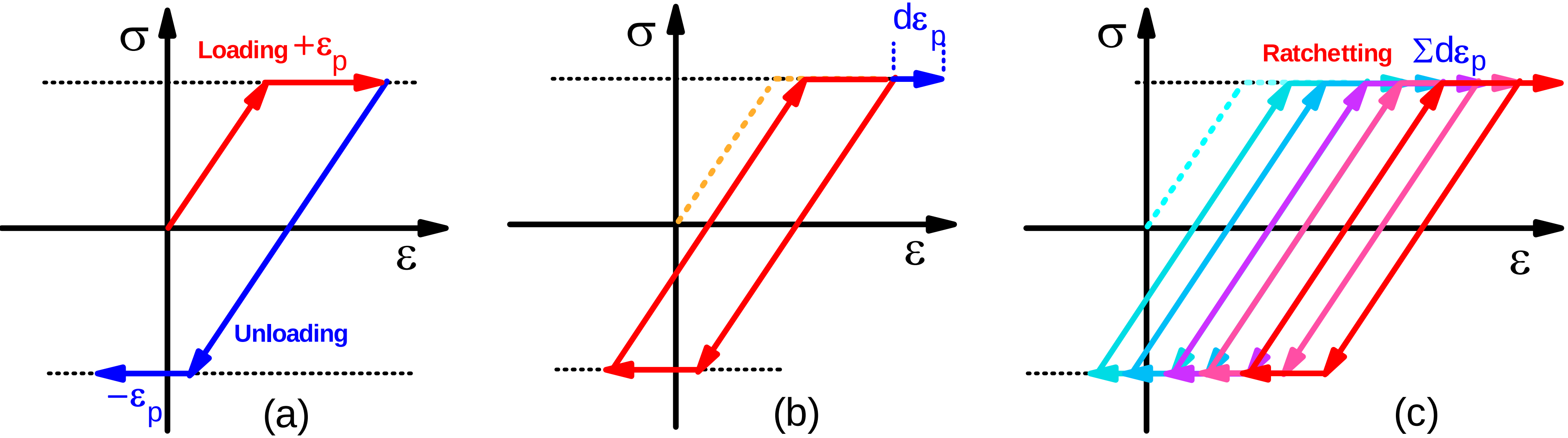
Plastic Shakedown



Elastic-perfectly plastic material

- Unloading plastic strain equal and opposite to loading plastic strain
  - Causes local damage and local failure mechanism
    - Low cycle fatigue associated with plastic strain range, $\Delta\ \mathcal{E}p$

Ratchetting

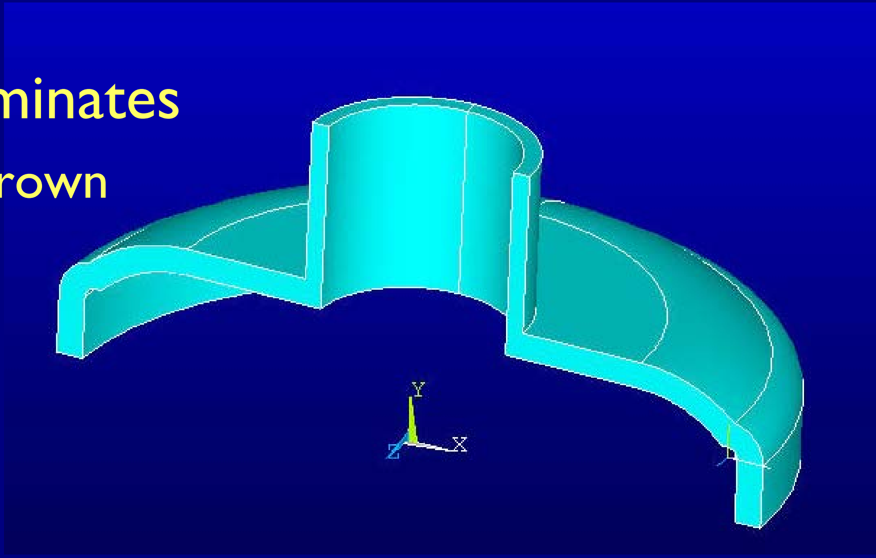


- Net plastic strain increment $d\mathcal{E}_p$ for each cycle
  - Accumulates $\Sigma d\mathcal{E}p$ to form global failure mechanism
    - Ratchetting or Incremental plastic collapse
- Ratchetting is not permitted in pressure vessel design

Analysis Methods

- The Codes do not specify use of specific analysis methods in design
  - In practice the terminology used suggests that the expected method is shell discontinuity analysis
    - State of the art in the 1960s

- Finite Element Analysis now predominates
  - The analysis capability has greatly outgrown PVD Code requirements
  - Problems can arise in interpreting code guidelines

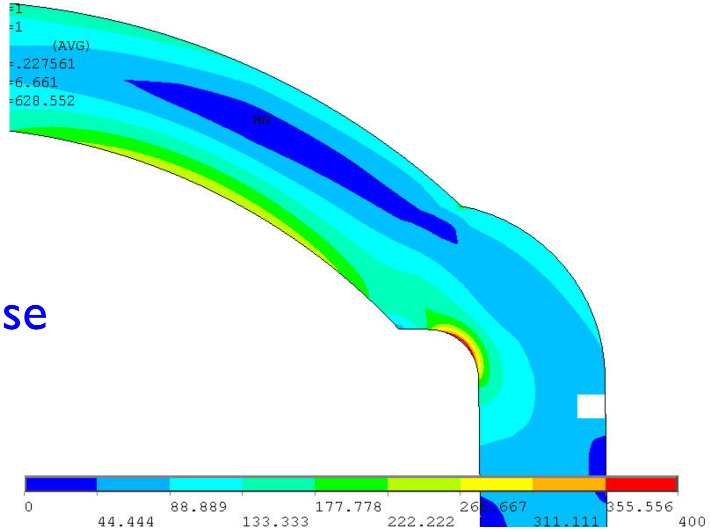


Elastic & Plastic Analysis

- Most Code guidelines are for elastic analysis
  - Problematic as gross plastic deformation and incremental Plastic collapse are inelastic failure mechanisms
- The Codes give limited guidelines for plastic analysis
  - Mostly limit analysis

Elastic Analysis & Stress Categorisation

- Elastic stress is categorised into three classes:
  - Primary stress
    - Associated with gross plastic deformation
  - Secondary stress (plus primary stress)
    - Associated with incremental plastic collapse
  - Peak stress (plus Primary plus secondary)
    - Associated with fatigue failure



- Each “type” of stresses is limited to different allowable values
  - Specified in terms of a design stress

Stress Classification

- Difficult and often subjective
  - Limited guidelines for specific configurations given in Codes
- Categories defined in terms of membrane and bending distributions

Stress Classification (PD5500)	Allowable Stress Intensity (wrt Design Stress f)	Allowable Stress Intensity
General primary membrane f_m	f	$2/3\ \sigma_y$
Local primary membrane f_L	$1.5\ f$	σ_y
Primary membrane plus bending $(f_m + f_b)$ or $(f_L + f_b)$	$1.5\ f$	σ_y
Primary plus secondary $(f_m + f_B + f_q Q)$ or $(f_L + f_b + f_q\ )$	$3\ f$	$2\ \sigma_y$

Stress Limits

- Primary stress
  - Yield limited to prevent gross plastic deformation
- Primary plus secondary
  - Limited to twice yield to assure shakedown
- Primary plus secondary plus peak
  - Determines fatigue design life

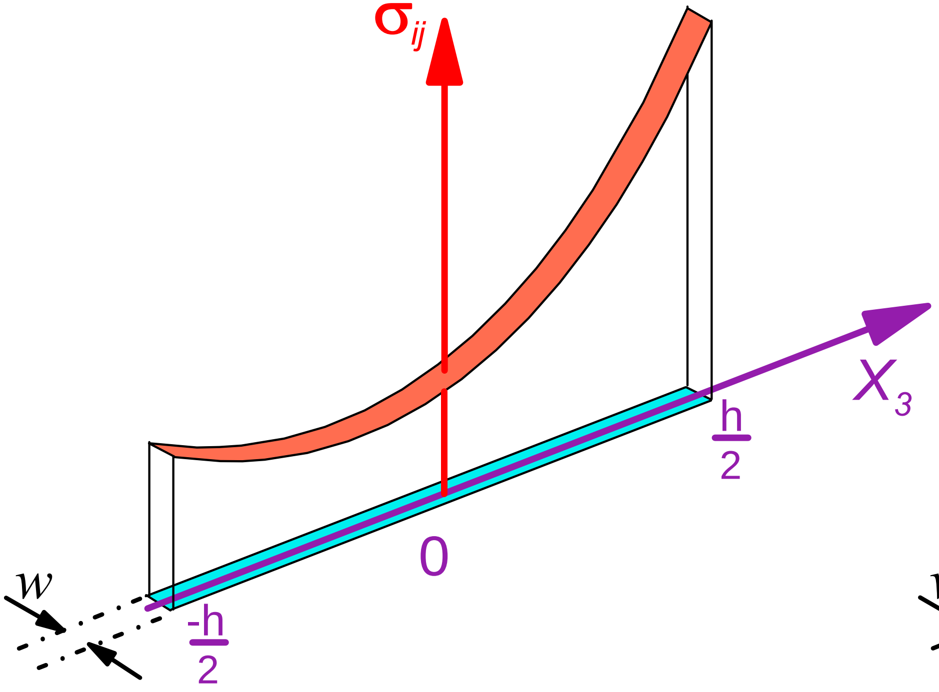
Shell FEA

- Can be used to model thin-shell vessels
  - Results given as membrane and bending stress
    - Compatible with Code categories
- Cannot be used for
  - Configurations with significant non-linear variation in through-thickness stress (thick vessels)
  - Complex 2-D or 3-D geometry

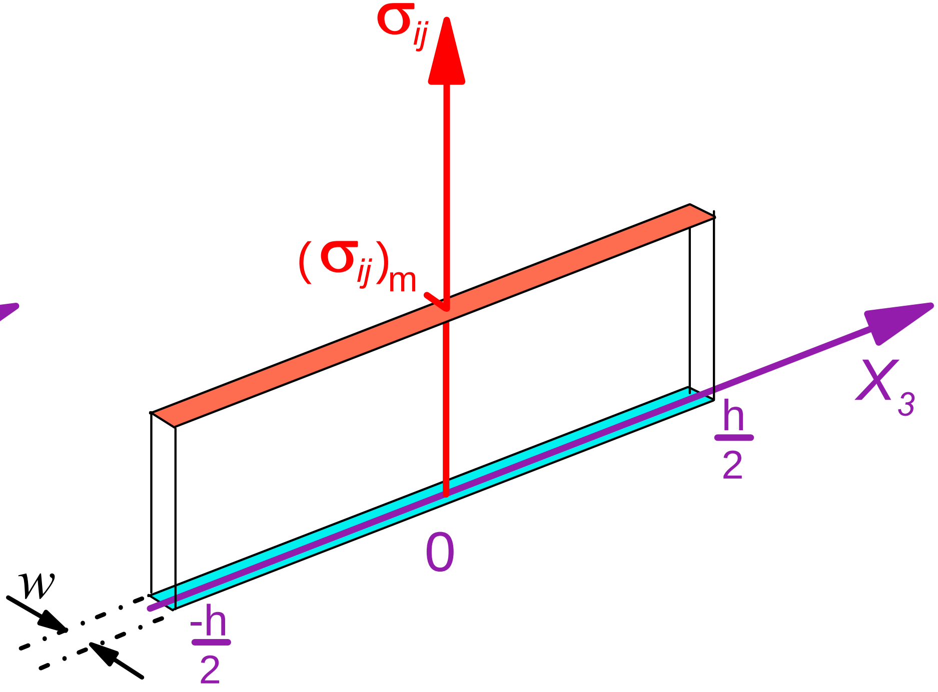
Solid FEA

- Can be used to model any 2-D or 3-D configuration
  - The total elastic stress at a point is defined by six elementary stresses σ_{ij}
    - Not compatible with Code membrane and bending stress categories
- A methodology is required to relate the distribution of to σ_{ij} *the code categories*
  - Most widely used method is through-thickness Stress Linearisation
    - Extracts pseudo membrane and bending stress from elastic distribution

Linearising Membrane Stress

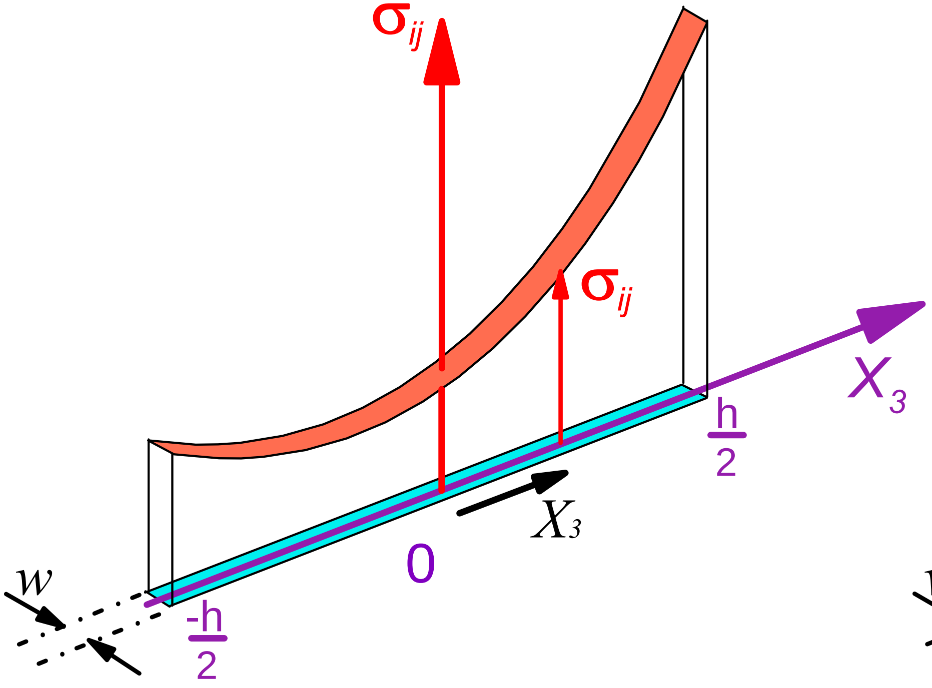


Nonlinear Stress

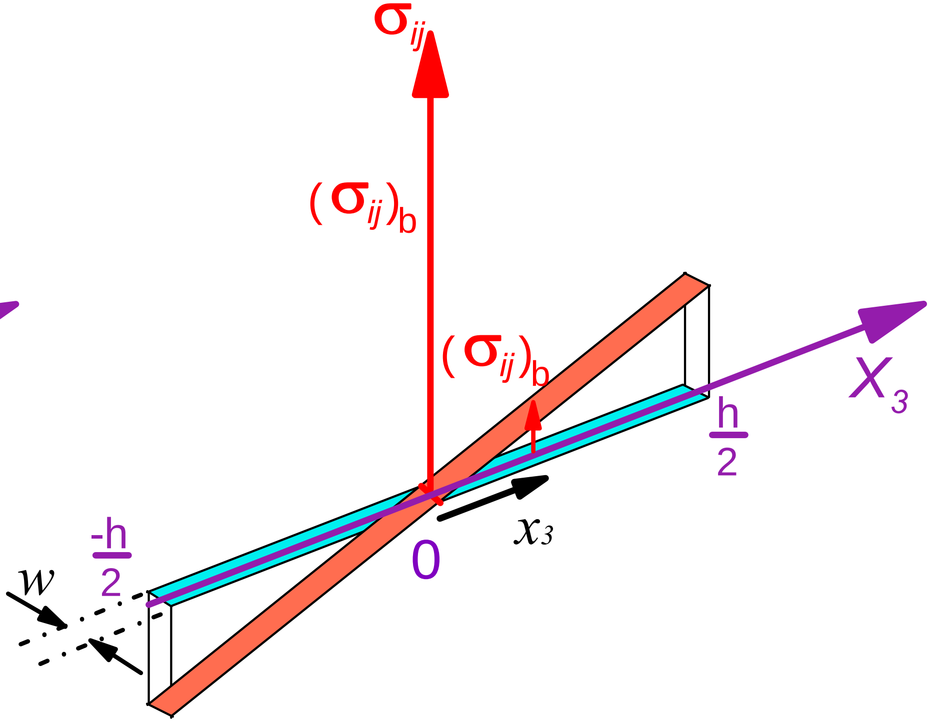


Equivalent Membrane Stress

Linearising Bending Stress



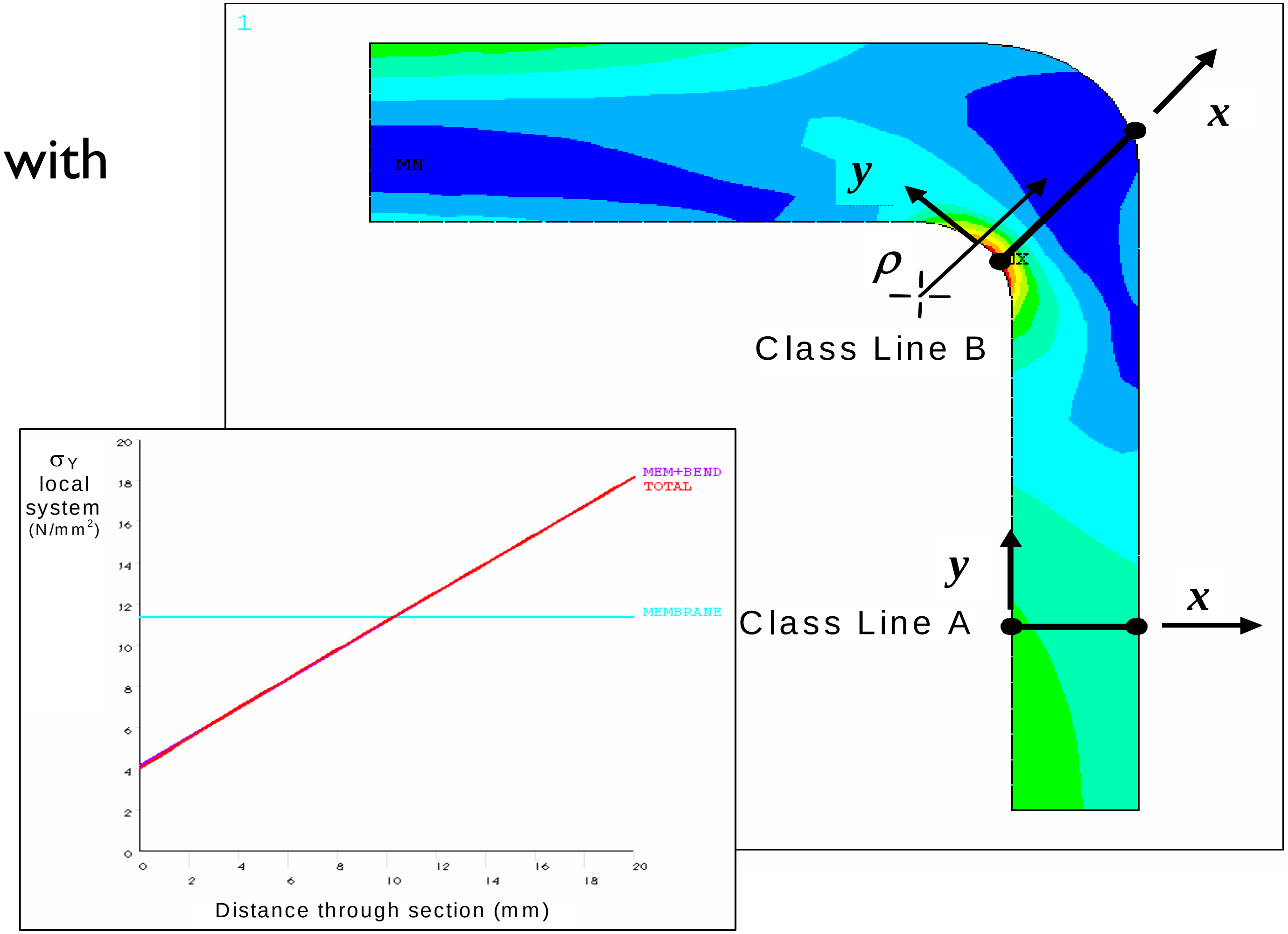
Nonlinear Stress



Equivalent Bending Stress

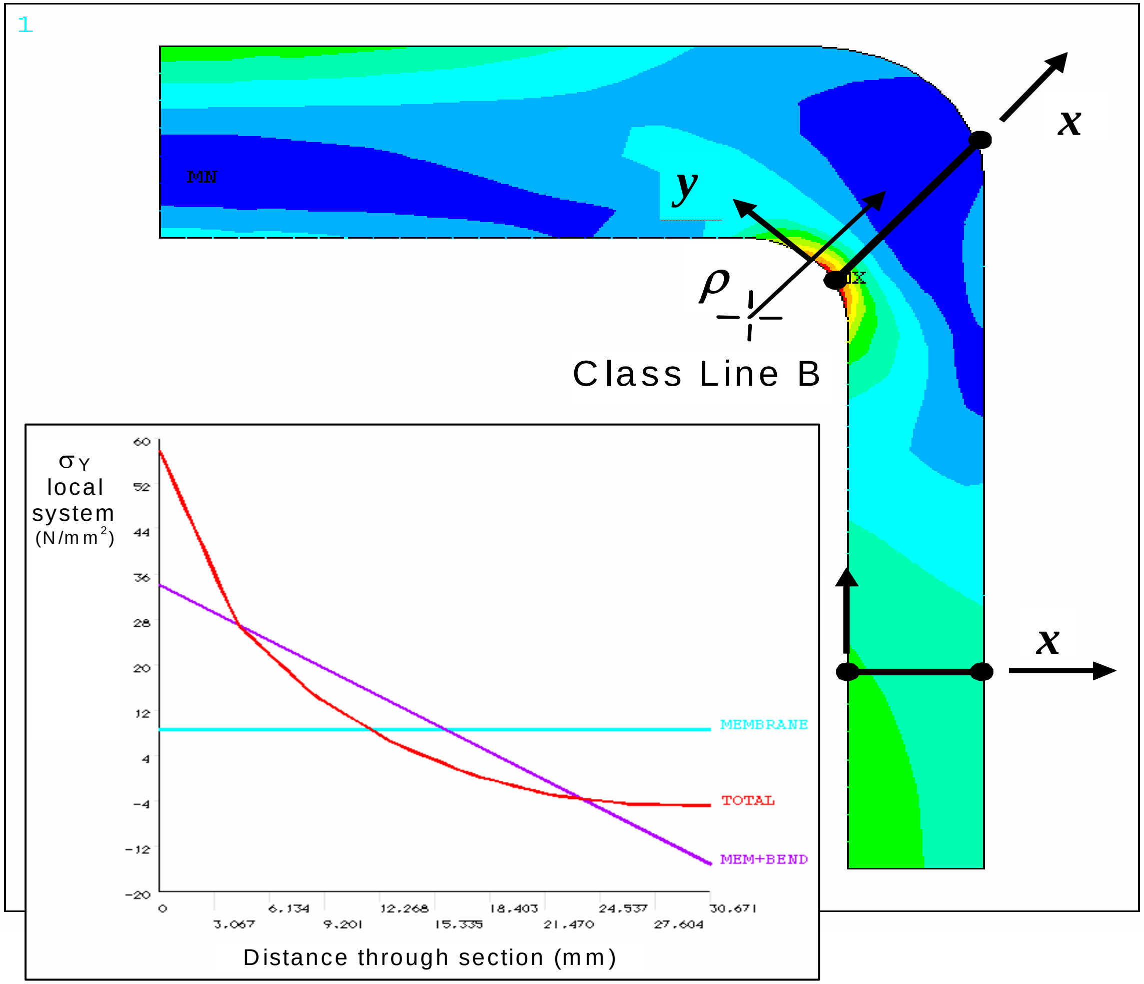
Linearisation Example

- Axisymmetric cylindrical shell with flat head
- Class Line A axial stress
  - “Reasonable” linearisation?



Linearisation Example

- Class Line B axial stress
  - “Reasonable” linearisation?
- How should the difference between total and mem+bend stress be treated?
  - Is it a peak stress?
  - If so, where is the associated SCF?



Limitations of Elastic Analysis

- The major problem is stress categorisation
  - Stress distribution must be defined in terms of membrane, bending and peak stress.
  - There is no theoretical justification for linearising certain regions of a vessel
    - Many papers have been published on the subject, more or less concluding that:
      - It is appropriate to place a class line anywhere in a vessel except where you would actually like to place one
        - » i.e. where the stress distribution is complex
  - In practice, designers usually regard linearisation as valid throughout the vessel, for both 2-D and 3-D models

Barriers to Elastic FEA

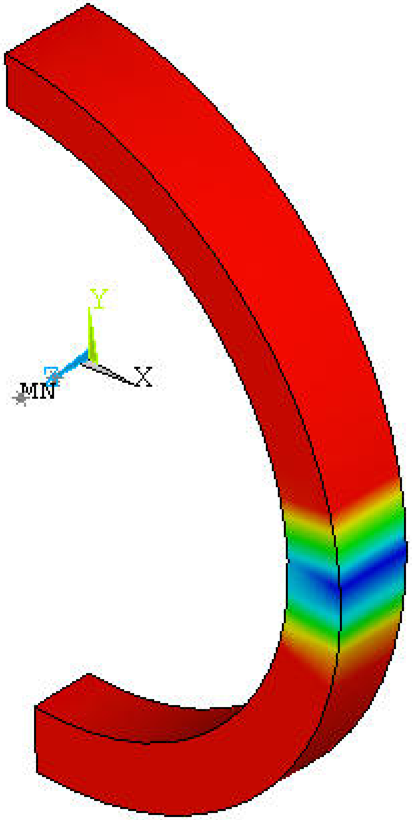
- The major problem is stress categorisation
  - 1997 survey of the European pressure vessel industry (as part of the JRC/EPERC *Design by Analysis* project) identified interpretation of finite element analysis results as the major problem in design by analysis.
  - The designer is required to:
    - Obtain linearised membrane and bending stresses
    - Assign these to the appropriate category
      - Primary – yield limited, possible over-conservative
      - Secondary – twice yield limited, possible not conservative
  - Code guidance is limited

Inelastic Analysis

- Inelastic analysis removes the stress categorisation problem
  - The analysis is related directly to inelastic failure mechanisms
    - Gross plastic deformation
    - Ratchetting
- Routine inelastic FEA capabilities have now overtaken DBA guidelines
  - Different types of inelastic analysis are included in the different Codes

PD5500 Inelastic DBA

- Considers Limit Analysis for gross plastic deformation check
  - Elastic-perfectly plastic material model
  - Small deformation (linear geometry) theory
    - Analysis evaluates the Limit Load
      - The allowable load is 2/3 the limit load
      - Relatively simple to perform
      - Requires only Code material data
- May be applied to non-standard configurations



EN13445 Inelastic DBA

- Considers Limit analysis but *Annex B Design by Analysis – Direct Route* [2] states:

  "In checks on structures … where deformation … has an unfavorable (weakening) effect, geometrically non-linear effects shall be taken into account in gross plastic deformation …checks".

    - Accounts for geometric weakening

  - No rules or procedures given for including material strain hardening
    - Enhances load carrying capacity

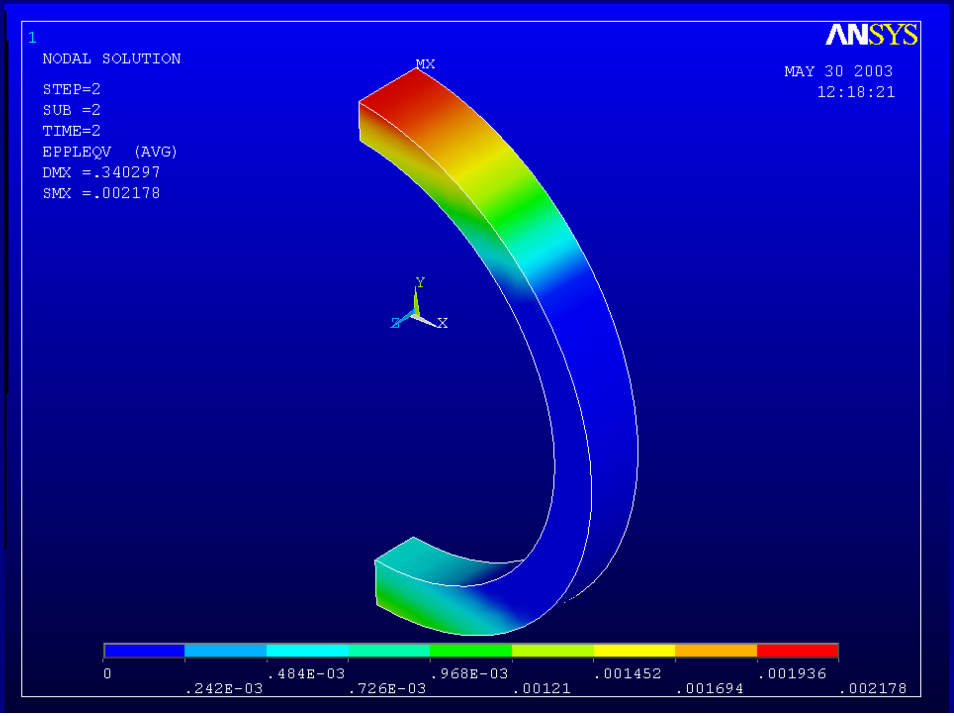
- May be applied to any configuration

ASME VIII Inelastic DBA

- Limit analysis procedures similar to PD5500
- Geometric non-linearity and strain hardening effects may be incorporated in a “Plastic Analysis”
- Defines the “plastic load” by application of a “criterion of plastic collapse”
  - The allowable load is 2/3 the plastic load
    - Plastic criterion dependent
      - ASME Twice Elastic Slope criterion has been shown to be inconsistent
      - Often results in plastic load similar to limit load

Shakedown

- Shakedown analysis procedures are limited and not generally well understood
- Inelastic shakedown analysis is permissible but not widely used
  - Incremental and bounding theorem approaches



Education & Training Requirements

- PD5500 and ASME VIII do not stipulate any particular education or training requirements for the designer
- EN13445 B and EN13445 C both stipulate that when using advanced analysis techniques:
  - "Due to the advanced methods applied, until sufficient in-house experience can be demonstrated, the involvement of an independent body, appropriately qualified in the field of DBA, is required in the assessment of the design (calculations) and the potential definition of particular NDT requirements."

Education & Training Requirements

- The Design by Analysis Manual (European Commission, DG-JRC/IAM, Petten, The Netherlands, 1999)
  Contains an extensive account of PV DBA
- An e-version is available on-line and free at:

http://ped.eurodyn.com/jrc/design_by_analysis

Use of DBA in Industry

- DBA tends to be used only when DBF is not possible
  - Non-standard configurations
  - Problem designs that do not satisfy DBF
- Tends to be restricted to organisations manufacturing advanced, high capitol cost designs
  - Wider use limited by
    - Cost of computer hardware, software and in-house expertise

      or
    - Cost of qualified external consultancy

Conclusion

- FE based DBA is long established in PVD
- Procedures were innovative when implemented but now lag routine analysis capabilities

- Advanced analysis methods simplify the design process
- Code guidelines need to be improved for
  - Inelastic analysis including large deformation and strain hardening effects
  - Shakedown analysis
- Adopting advanced analysis methods has implications for Education & Training of designers